Number systems: Non-positional number systems and Positional number systems

Introduction

A number system (or numeral system) defines how a number can be represented using distinct symbols. A number can be represented differently in different systems. Several number systems have been used in the past and can be categorized into two groups: positional and non-positional systems. Our main goal is to discuss the positional number systems, but we also give examples of non-positional systems.

1- POSITIONAL NUMBER SYSTEMS

In a positional number system, the position a symbol occupies in the number determines the value it represents. In this system, a number represented as:

$$\pm (S_{K-1} \ldots S_2 S_1 S_0 . S_{-1} S_{-2} \ldots S_{-L})_b$$

has the value of:

$$n = \pm \; S_{K-1} \times b^{K-1} + \ldots + S_1 \times b^1 + S_0 \times b^0$$
$$+ \; S_{-1} \times b^{-1} + S_{-2} \times b^{-2} + \ldots + S_{-L} \times b^{-L}$$

in which S is the set of symbols, b is the base (or radix), which is equal to the total number of the symbols in the set S, and S_K and S_L are symbols in the whole and fraction parts of the number.

1.1 The decimal system (base 10)

The first positional number system we discuss in this chapter is the decimal system. The word decimal is derived from the Latin root *decem* (ten). In this system the base b =10 and we use ten symbols to represent a number. The set of symbols is S = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}. As we know, the symbols in this system are often referred to as decimal digits or just digits. In the decimal system, a number is written as:

$$\pm (S_{K-1} \ldots S_2 S_1 S_0 . S_{-1} S_{-2} \ldots S_{-L})_{10}$$

Integers

An integer (an integral number with no fractional part) in the decimal system is familiar to all of us—we use integers in our daily life. The value is calculated as:

$$N = \pm \; S_{K-1} \times 10^{K-1} + S_{K-2} \times 10^{K-2} + \ldots + S_2 \times 10^2 + S_1 \times 10^1 + S_0 \times 10^0$$

Example1// The following shows the place values for the integer +224 in the decimal system:

			10^2		10^1		10^0	Place values
			2		2		4	Number
N	=	+	2×10^2	+	2×10^1	+	4×10^0	Values

Reals

A real (a number with a fractional part) in the decimal system is also familiar. For example, we use this system to show dollars and cents ($23.40). The value is calculated as:

$$R = \pm \underbrace{S_{K-1} \times 10^{K-1} + \ldots + S_1 \times 10^1 + S_0 \times 10^0}_{\text{Integral part}} + \underbrace{S_{-1} \times 10^{-1} + \ldots + S_{-L} \times 10^{-L}}_{\text{Fractional part}}$$

Example 2// The following shows the place values for the real number +24.13:

	10^1		10^0		10^{-1}		10^{-2}	Place values
	2		4	•	1		3	Number
$R = +$	2×10	+	4×1	+	1×0.1	+	3×0.01	Values

1.2 The binary system (base 2)

The second positional number system we discuss in this chapter is the binary system. The word binary is derived from the Latin root *bini* (or two by two). In this system the base b = 2 and we use only two symbols, S = {0, 1}. The symbols in this system are often referred to as binary digits or bits (binary digit).

Integers

We can represent an integer as +_ $(S_{K-1} \ldots S_1 S_0)_2$. The value is calculated as:

Example3// The following shows that the number $(11001)_2$ in binary is the same as 25 in decimal. The subscript 2 shows that the base is 2:

	2^4		2^3		2^2		2^1		2^0	Place values
	1		1		0		0		1	Number
$N =$	1×2^4	+	1×2^3	+	0×2^2	+	0×2^1	+	1×2^0	Decimal

Note that the equivalent decimal number is $N = 16 + 8 + 0 + 0 + 1 = 25$.

Reals

A real—a number with an optional fractional part—in the binary system can be made of K bits on the left and L bits on the right, +_ $(S_{K-1} \ldots S_1\ S_0 \cdot S_{-1} \ldots S_{-L})_2$. The value can be calculated as:

$$R = \pm \underbrace{S_{K-1} \times 2^{K-1} \times \ldots \times S_1 \times 2^1 \times S_0 \times 2^0}_{\text{Integral part}} + \bullet \underbrace{S_{-1} \times 2^{-1} + \ldots + S_{-L} \times 2^{-L}}_{\text{Fractional part}}$$

Example 4// The following shows that the number $(101.11)_2$ in binary is equal to the number 5.75 in decimal:

	2^2		2^1		2^0		2^{-1}		2^{-2}	Place values
	1		0		1	•	1		1	Number
$R =$	1×2^2	+	0×2^1	+	1×2^0	+	1×2^{-1}	+	1×2^{-2}	Values

Note that the value in the decimal system is $R = 4 + 0 + 1 + 0.5 + 0.25 = 5.75$.

1.3 The hexadecimal system (base 16)

The word hexadecimal is derived from the Greek root hex (six) and the Latin root *decem* (ten). To be consistent with decimal and binary, it should really have been called sexadecimal, from the Latin roots sex and decem. In this system the base b 5 16 and we use 16 symbols to represent a number. The set of symbols is S = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F}. Note that the symbols A, B, C, D, E, F (uppercase or lowercase) are equivalent to 10, 11, 12, 13, 14, and 15 respectively. The symbols in this system are often referred to as hexadecimal digits.

Integer

Example 5// The following shows that the number $(2AE)_{16}$ in hexadecimal is equivalent to 686 in decimal:

	16^2		16^1		16^0	Place values
	2		A		E	Number
$N =$	2×16^2	+	10×16^1	+	14×16^0	Values

Note that the value in the decimal system is $N = 512 + 160 + 14 = 686$.

Real

Although a real number can be also represented in the hexadecimal system, it is not very common.

1.4 The octal system (base 8)

The second system that was devised to show the equivalent of the binary system outside the computer is the octal system. The word octal is derived from the Latin root *octo* (eight). In this system the base b 5 8 and we use eight symbols to represent a number. The set of symbols is S = {0, 1, 2, 3, 4, 5, 6, 7}. The symbols in this system are often referred to as octal digits.

Integer

Example 6// The following shows that the number $(1256)_8$ in octal is the same as 686 in decimal:

	8^3		8^2		8^1		8^0	Place values
	1		2		5		6	Number
$N =$	1×8^3	+	2×8^2	+	5×8^1	+	6×8^0	Values

Note that the decimal number is $N = 512 + 128 + 40 + 6 = 686$.

Reals

Although a real number can be also represented in the octal system, it is not very common.

Summary of the four positional systems

System	Base	Symbols	Examples
Decimal	10	0, 1, 2, 3, 4, 5, 6, 7, 8, 9	2345.56
Binary	2	0, 1	$(1001.11)_2$
Octal	8	0, 1, 2, 3, 4, 5, 6, 7	$(156.23)_8$
Hexadecimal	16	0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F	$(A2C.A1)_{16}$

2- NONPOSITIONAL NUMBER SYSTEMS

Although nonpositional number systems are not used in computers, we give a short review here for comparison with positional number systems. A nonpositional number system still uses a limited number of symbols in which each symbol has a value. However, the position a symbol occupies in the number normally bears no relation to its value—the value of each symbol is fixed. To find the value of a number, we add the value of all symbols present in the representation.

Example7 //The Roman number system is a good example of a nonpositional number system. This system was invented by the Romans and was used until the sixteenth century in Europe. Roman numerals are still used in sports events, clock dials, and other applications. This number system has a set of symbols S = {I, V, X, L, C, D, M}. The values of each symbol are shown

Symbol	I	V	X	L	C	D	M
Value	1	5	10	50	100	500	1000

Some example:-

III	→	1 + 1 + 1	=	3
IV	→	5 − 1	=	4
VIII	→	5 + 1 + 1 + 1	=	8
XVIII	→	10 + 5 + 1 + 1 + 1	=	18
XIX	→	10 + (10 − 1)	=	19
LXXII	→	50 + 10 + 10 + 1 + 1	=	72
CI	→	100 + 1	=	101
MMVII	→	1000 + 1000 + 5 + 1 + 1	=	2007
MDC	→	1000 + 500 + 100	=	1600